

Stage di ANPOBASSO - 2012

Titolo nota

24/02/2012

Algebra

•) $x^4 + \dots + 12$ radici intere
 \uparrow
 ax^3

$$12 = x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot \begin{cases} 12 \cdot 1 \cdot 1 \cdot 1 \rightarrow 15 \\ -12, -1, -1, -1 \rightarrow -15 \\ -12, -1, 1, 1 \rightarrow -11 \\ 12, -1, -1, 1 \rightarrow 11 \end{cases} \quad 4$$

$$\begin{cases} -1, -2, -2, -3 \rightarrow -8 \\ 1, 2, 2, 3 \rightarrow 8 \\ -1, -2, 2, 3 \rightarrow 2 \\ -1, 2, 2, -3 \rightarrow 0 \\ 1, 2, -2, -3 \rightarrow -2 \\ 1, -2, -2, 3 \rightarrow 0 \end{cases} \quad 5 \quad \begin{cases} 4, 3, 1, 1 \rightarrow 9 \\ -4, -3, -1, -1 \rightarrow -9 \\ -4, -3, 1, 1 \rightarrow -5 \\ 4, 3, -1, -1 \rightarrow 5 \\ -4, 3, -1, 1 \rightarrow -1 \\ 4, -3, 1, -1 \rightarrow 1 \end{cases}$$

$$\begin{cases} 6, 2, 1, 1 \rightarrow 10 \\ -6, -2, -1, -1 \rightarrow -10 \\ 6, -2, 1, -1 \rightarrow 4 \\ -6, 2, -1, 1 \rightarrow -4 \\ -6, -2, 1, 1 \rightarrow -6 \\ 6, 2, -1, -1 \rightarrow 6 \end{cases} \quad \begin{aligned} 4 + 5 + 2 \cdot 6 = \\ = 9 + 12 = 21 \end{aligned}$$

•) $x^{2008} - x - 2008 = 0 \quad x \geq 0$

$$x^{4016} - 4016x = ?$$

$$x^{2008} = x + 2008 > x \Rightarrow \text{la radice } \bar{e} > 1$$

$$2^{2008} \neq 2010$$

$$1 < x < 2$$

$$1 < x < \sqrt{2}$$

$$x^{4016} = (x^{2008})^2 = (x + 2008)^2 = x^2 + 4016x + 2008^2$$

$$x^{4016} - 4016x = \underbrace{x^2 + 2008^2}$$

↑ ↑

1, ...

2064 +

1, ... =

2065

•) $p(x) = x^{20} + \dots + a_0$ coeff. interi.

$$p(1) = 2 \quad p(2) = 4, \dots, \quad p(20) = 40$$

le ultime 3 cifre di $p(21)$

$$p(x) - 2x = q(x) \quad \text{monico di grado } 20.$$

$q(x)$ ha radici $1, 2, \dots, 20$

$$q(x) = (x-1)(x-2) \dots (x-20)$$

$$p(x) = (x-1)(x-2) \dots (x-20) + 2x$$

$$p(21) = 20 \cdot 19 \cdot 18 \cdot \dots \cdot 1 + 42 = 20! + 42$$

$$20 \cdot 10 \cdot 5 \cdot 15 \cdot 2 \cdot 4$$

$$10 \cdot 10 \cdot 5 \cdot 5 \cdot 2 \cdot 2 = 10 \cdot 10 \cdot 10 \cdot 10 = 10000$$

1042

$$P(x) = x^3 + ax^2 + bx + cx \quad \alpha, \beta, \gamma \quad \alpha + \beta = 0.$$

$$a = -(\underbrace{\alpha + \beta + \gamma}_0) = -\gamma$$

$$\alpha\beta\gamma = -\beta^2\gamma = -ab$$

$$b = \alpha\beta + \beta\gamma + \gamma\alpha = -\beta^2 - \beta\gamma$$

$$-c$$

$$c = ab \quad \textcircled{B}$$

$$p(x) = (1+x^2)(1+x^4)(1+x^8) \dots (1+x^{2^{10}})$$

$$p(10000) \quad 10^3 a + 10^2 b + 10c + d = q(10)$$

$$q(T) = Ta^3 + T^2 b + Tc + d$$

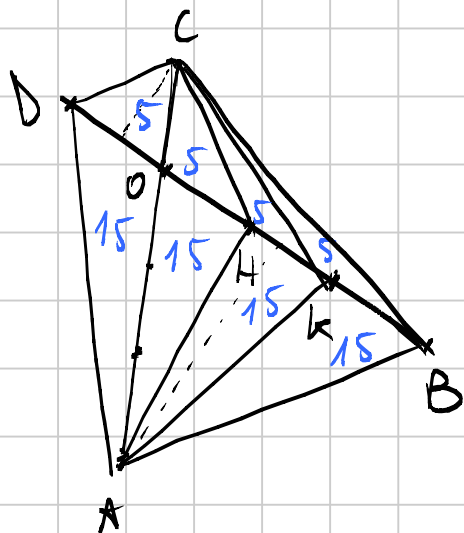
$$q(1) = a + b + c + d$$

$$p(2^3) = (1+T^6)(1+T^{12}) \dots (1+T^{3 \cdot 2^{10}})$$

$$p(1) = \underbrace{2 \cdot 2 \cdot 2 \cdot \dots \cdot 2}_{10} = 2^{10} \quad \boxed{1024}$$

es. GEOM.

③



$$R_{\triangle BOC} = 15$$

$$3 DO = OB$$

$$R_{\triangle BOC} = R_{\triangle OCH} + R_{\triangle CHK} + R_{\triangle CKB}$$

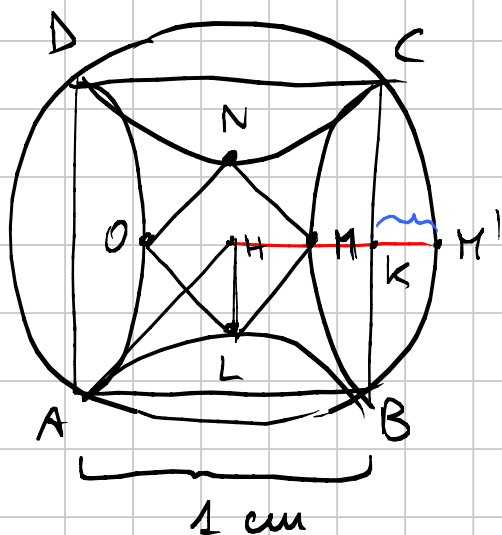
$$DO = HK = OH = KB$$

$$R_{\triangle OHC} = 5 \quad \leftarrow \quad 15 = R_{\triangle BOC} = 3 R_{\triangle OHC}$$

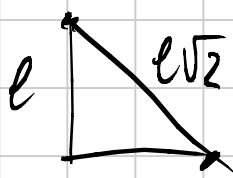
$$R_{\triangle COB} = 15 \quad \triangle COB = \triangle DOA \Rightarrow R_{\triangle DOA} = 15$$

$$R_{ABCD} = 15 \cdot 4 + 5 \cdot 4 = 60 + 20 = 80$$

②



R_{LMNO} ?



$$AC = \sqrt{2} \quad AH = \frac{\sqrt{2}}{2} = R$$

$$HM + MK = HK$$

$$HM = HK - MK = \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2}$$

$$HM' = \frac{\sqrt{2}}{2}$$

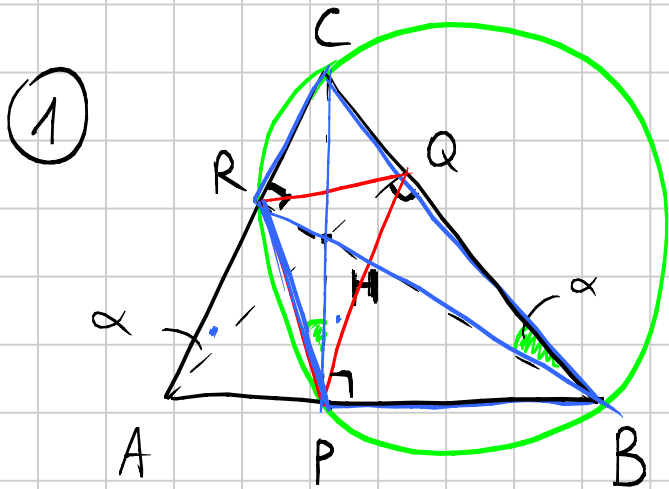
$$HK = \frac{1}{2}$$

$$\leftarrow MK = KM' = HM' - HK = \frac{1}{2}(\sqrt{2} - 1)$$

$$HM = 1 - \frac{\sqrt{2}}{2}$$

$$LM = HM \cdot \sqrt{2} = \sqrt{2} - 1$$

$$R_{LNNO} = (\sqrt{2} - 1)^2$$



$\triangle CRB$, $\triangle CPB$ tr. rettangoli
 (BC ipotenusa in comune)

PRCB CICLICO

$$\hat{R}PC = \hat{R}BC \quad (\text{ang. circ})$$

su \widehat{RC}

Ripeto su CQPA $\hat{C}PQ = \hat{C}AQ$

CP bisettrice di $\hat{R}PQ$ H incentro $\hat{R}PQ$